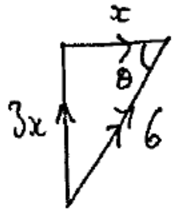
M1 June 2009 Solutions (OCR 4728)

(1)



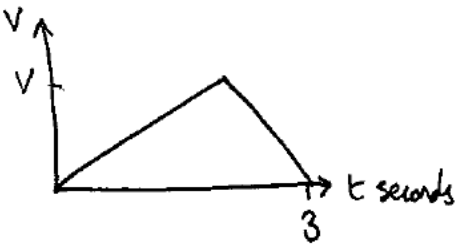
i) Use Pythagoras:

$x^2 + (3x)^2 = 6^2$
$x^2 + 9x^2 = 36$
$10x^2 = 36$
$x^2 = 3.6$ $\quad$ $\underline{\underline{x = 1.90}}$

ii) Angle θ between resultant & small force:

$\tan\theta = \frac{3x}{x}$ $\quad$ $\theta = \tan^{-1}(3) = \underline{\underline{71.6°}}$

(2) i)



ii) Area under graph = 6

Area of Δ = $\frac{1}{2}bh$
$= \frac{1}{2} \times 3 \times V$

So $6 = \frac{1}{2} \times 3 \times V$

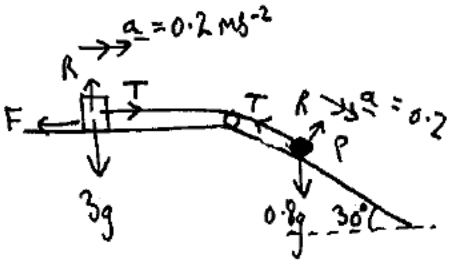
$V = \frac{6 \times 2}{3} = \underline{\underline{4\ ms^{-1}}}$

iii) acceleration is $2.4\ ms^{-2}$

Gradient $= \frac{4}{t} = 2.4$ $\quad$ So $t = \frac{4}{2.4} = 1\frac{2}{3}$ secs

So car is decelerating for $1\frac{1}{3}$ secs $\quad$ Deceleration $= 4/1\frac{1}{3} = \underline{\underline{3\ ms^{-2}}}$

(3)



i) Show tension in string is 3.76N:

P is accelerating down slope, and Force causing this acceleration is given by "F = Ma"

Resolve down slope: $0.8g\cos 60 - T = Ma$

$T = 0.8g\cos 60 - Ma$
$= 0.4g - 0.8 \times 0.2$
$= \underline{\underline{3.76N}}$

ii) Want μ for B:

Equation of motion for B: $T - F = Ma$

$3.76 - F = 3 \times 0.2$

Frictional force $F = 3.76 - 3 \times 0.2 = 3.16$

$F = \mu R$

Resolve vertically: $R = 3g$

Then $F = \mu 3g$

Sub 3.16 for F: $\quad$ $3.16 = \mu \times 3 \times 9.8$

$\mu = \frac{3.16}{(3 \times 9.8)} = \underline{\underline{0.107}}$

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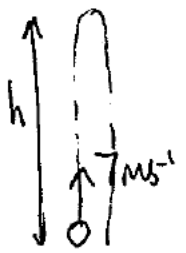
(4)



Acceleration due to gravity $g = 9.8ms^{-2}$.

i) $+\uparrow$
$s = 2.1$
$u = 7$
$v = $ want
$a = -9.8$

$v^2 = u^2 + 2as$
$= 7^2 - 2 \times 9.8 \times 2.1$
$= 7.84$
$\underline{\underline{v = 2.8ms^{-1}}}$ at a height of 2.1

ii) <u>Greatest height reached:</u>

Consider motion up to greatest height (when $v = 0$)

$+\uparrow$
$s = h$
$u = 7$
$v = 0$
$a = -9.8$

$v^2 = u^2 + 2as$
$0 = 7^2 - 2 \times 9.8 \times h$
$0 = 49 - 19.6h$
$19.6h = 49$
$h = \frac{49}{19.6} = \underline{\underline{2.5m}}$

iii) <u>Time when travelling down at $v = 5.7$:</u>

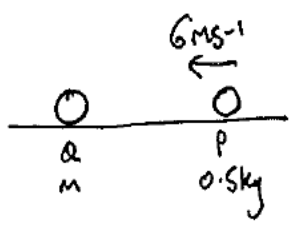
$+\uparrow$
s
$u = 7$
$v = -5.7$
$a = -9.8$
$t = $ want

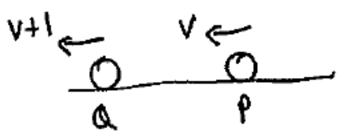
$v = u + at$
$t = \frac{v-u}{a}$
$t = \frac{-5.7-7}{-9.8} = \frac{-12.7}{-9.8} = 1.2959$

<u>time = 1.30 secs</u>

(5) Before collision After:

i)

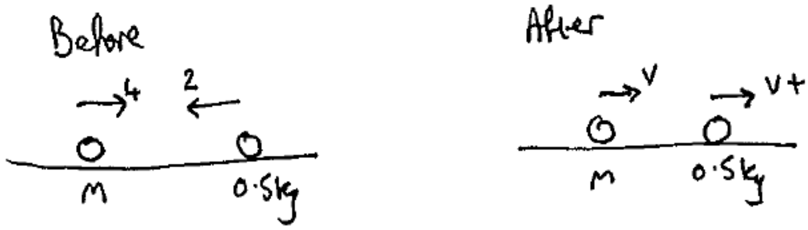




Conservation of Momentum $+\leftarrow$

$0 + (0.5 \times 6) = m(v+1) + 0.5v$
$3 = mv + m + 0.5v$
$-m + 3 = mv + 0.5v$
$\underline{-m + 3 = v(m + 0.5)}$

⑤ ii)



Conserve momentum $\rightarrow +$

$$4m - (0.5\times 2) = mv + 0.5(v+1)$$
$$4m - 1 = mv + 0.5v + 0.5$$
$$\underline{\underline{4m - 1.5 = v(m+0.5)}} \qquad a=4,\ b=-1.5$$

iii) Show $m = 0.9$ & find v

$$-m+3 = v(m+0.5) \quad (1)$$
$$4m-1.5 = v(m+0.5) \quad (2)$$

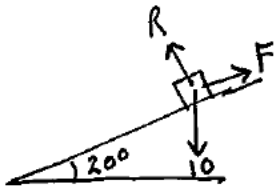
Put LHS (1) = (2)

$$-m+3 = 4m-1.5$$
$$3 = 5m - 1.5$$
$$4.5 = 5m$$
$$m = \frac{4.5}{5} = \underline{\underline{0.9}}$$

Substitute for m into (2):

$$(4\times 0.9) - 1.5 = v(0.9+0.5)$$
$$2.1 = 1.4v$$
$$v = \frac{2.1}{1.4} = \underline{\underline{1.5\ \text{ms}^{-1}}}$$

⑥



Constant speed so no resultant force, and friction has its maximum value $F=\mu R$.

Contact force is R&F :

i)a) Resolve ⊥ plane : $R = 10\cos 20° = \underline{9.40\ \text{N}}$ perpendicular

Resolve // plane : $F = 10\cos 70° = \underline{3.42\ \text{N}}$ parallel

b) $F = \mu R$

So $\mu = \frac{F}{R} = \frac{3.42}{9.40} = \underline{\underline{0.364}}$

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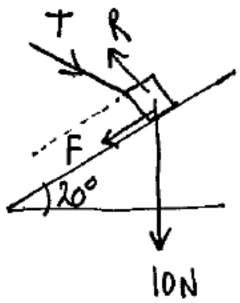
⑥ ii)



Limiting equilibrium so $F = \mu R = 0.364R$

Resolve // plane: $F + 10\cos 70° = T\cos 45$ ①

Resolve ⊥ plane: $R = 10\cos 20° + T\cos 45$ ②

① $0.364R = T\cos 45 - 10\cos 70°$

② $0.364R = 0.364(10\cos 20 + T\cos 45)$

Put equal $T\cos 45 - 10\cos 70° = 0.364 \times 10\cos 20 + 0.364T\cos 45$

$T(\cos 45 - 0.364\cos 45) = 10\cos 70 + 3.64\cos 20$

$$T = \frac{10\cos 70 + 3.64\cos 20}{(\cos 45 - 0.364\cos 45)}$$

$= 15.2\text{N}$

⑦

i) Want $\underset{\sim}{a}$ in 1st 3 seconds:

$\underset{\sim}{V} = 6t - t^2$

$\underset{\sim}{a} = \frac{dV}{dt} = 6 - 2t \text{ ms}^{-2}$

ii) Show S runs 18m in 1st 3 seconds:

$$S = \int v\,dt = \int_0^3 6t - t^2\,dt = \left[\frac{6t^2}{2} - \frac{t^3}{3}\right]_0^3 = \left(3(3^2) - \frac{3^3}{3}\right) - 0$$

$= 18\text{m}$

iii) Time taken to run 100m:

$S = 18\text{m}$ at 3 secs, So 82m left

At 9ms^{-1} constant speed time $= \frac{82}{9} = 9.11$ secs

Total time to run 100m = $3 + 9.1 = 12.1$ secs

iv) Time to run 200m:

18m in 1st 3 secons

$19 \times 9 = 171\text{ m}$ from $t = 3$ to $t = 22$ - Need 11m more while decelerating

$S = 11$
$u = 9$
$a = -0.6$

$S = ut + \frac{1}{2}at^2$
$11 = 9t - 0.3t^2$
$0.3t^2 - 9t + 11 = 0$

$$t = \frac{9 \pm \sqrt{9^2 - 4 \times 0.3 \times 11}}{0.6} = 1.28$$

Total time = 23.3 secs